Effects of coupling geometries on the multi-mode open Dicke model

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In the open Dicke model, which describes the collective coupling of N two level atoms to a single cavity mode, there is a well known phase transition in the steady-state behaviour of the system, as the light-matter coupling strength is varied, from a normal phase to a superradiant phase [1,2]. Multi-mode versions of this model, where several distinct clusters of N atoms are coupled to the degenerate modes of a confocal cavity, have been studied in the context of spin glass [3] and associative memory regimes [4], for which the couplings between different clusters of atoms and different modes are chosen effectively randomly, without the choice of a particular geometry.

In this theoretical work, we are investigating the effects of specific coupling geometries on the multi-mode Dicke model and its solutions. Even in the simplest possible, effectively 1D nearest neighbour geometry, in which there are M clusters of atoms and $M-1$ cavity modes arranged so that each cavity mode couples to two neighbouring clusters of atoms, we observe effects not seen in the standard, single-mode Dicke model. We show the existence of additional stable steady-state solutions beyond the superradiant transition, the number of which increases with the number of atom clusters in the system, and make predictions about their structure. We also investigate the dynamics of the system in the simplest configuration where this new behaviour is observed; a system composed of four clusters of two-level atoms coupled to three cavity modes.

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